

The x -dependence of Parton Distributions Compared with Neutrino Data

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We use the variational principle of Quantum HadronDynamics, an alternative formulation of Quantum ChromoDynamics, to determine the wavefunction of valence quarks in a baryon at a low value of Q^2 . This can be used to predict the structure function $x F_3(x, Q^2)$ at higher values of Q^2 using the evolution equations of perturbative QCD. This prediction is compared to the measurements of neutrino scattering cross-section by the CDHS and CCFR experiments. The agreement is quite good, confirming the validity of QHD as a way of studying hadronic structure.

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One of us has proposed [1] that it is possible to describe strong interactions directly in terms of hadrons rather than in terms of quarks and gluons; this new approach was called Quantum HadronDynamics (QHD). It is equivalent to Quantum ChromoDynamics (QCD, the universally accepted theory of strong interactions), except that the semi-classical approximation of QHD is a good approximation to nature: it is related to the large N_c limit of QCD. Semi-classical methods applied directly to QCD give results that are too inaccurate except in the limit of short distances. QHD has been constructed so far only in two space-time dimensions. However, that is sufficient to obtain the structure functions of Deep Inelastic Scattering: in the limit of zero transverse momentum for the constituents, we can dimensionally reduce the theory to two space-time dimensions. More precisely, the dependence of the structure functions on the Bjorken variable is determined by ignoring the transverse dimensions [2]. Thus we can predict the non-perturbative x_B dependence of structure functions, something that is completely inaccessible to the standard perturbative formulation of QCD. The leading order effects of transverse momenta are described by the DGLAP [3] equations that determine the Q^2 dependence of the structure functions: this is the fundamental insight of the paper of Altarelli and Parisi.

In this paper we will focus on understanding the distribution of valence quarks in the nucleon. This is measured directly in neutrino (and anti-neutrino scattering) against the nucleon: the isospin averaged valence parton distribution function is just the structure function $F_3(x_B)$ measured in this process [4]. To leading order in $\frac{1}{\log(Q^2)}$, it is not necessary to know the gluon structure functions of the hadron in the DGLAP evolution of this quantity. The anti-quark content of the baryon is zero at the initial value $Q^2 = Q_0^2$ within our approximations. This is consistent with the phenomenological

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model of Glück and Reya [6], that the anti-quark distribution of the proton is dynamically generated by Q^2 evolution from an initial value of zero at low Q^2 . We will, in a later paper [7], calculate the anti-quark distribution functions and show that this is indeed justified. However, we expect the “primordial” gluon distribution to be non-zero.

In a previous paper [8] we derived a variational principle that determines this valence quark distribution function in the semi-classical approximation of QHD. It was also shown there how to derive this variational principle from QCD. In a separate paper [9] we derived the same principle from an interacting valence parton model of the baryon. In this paper we will obtain approximate solutions of the variational principle and compare with the experimental data of the CCFR and CDHS [10] collaborations. The agreement of the predictions of QHD with data is quite spectacular: we now have a theory of the x_B dependence of deep inelastic structure functions.

Let $\tilde{\psi}(p)$ be the wavefunction of a valence parton in a proton, thought of as a function of the null component of momentum ($p = p^0 - p^1$) [1]. For kinematical reasons, $P > p > 0$, where P is the total momentum of the proton. Naively, we would have the sum rules,

$$\int_0^P |\tilde{\psi}(p)|^2 \frac{dp}{2\pi} = 1, \quad N_c \int_0^P p |\tilde{\psi}(p)|^2 \frac{dp}{2\pi} = P.$$

However, we have to modify the momentum sum rule since it is known that the valence partons carry only about half the momentum of the proton: the rest is carried mostly by the gluons, with a small contribution from anti-quarks. Thus we impose $N_c \int_0^P p |\tilde{\psi}(p)|^2 \frac{dp}{2\pi} = fP$, where f is the fraction of the momentum carried by the valence partons. We will address the problem of gluon distribution functions in a later paper, where we will attempt to calculate f ; here we will treat it as a parameter.

The wavefunction $\tilde{\psi}$ is determined by minimizing the energy

$$E_1(\psi) = \int_0^P \frac{1}{2} \left[p + \frac{\mu^2}{p} \right] |\tilde{\psi}(p)|^2 \frac{dp}{2\pi} + \frac{\tilde{g}^2}{2} \int |\psi(x)|^2 |\psi(y)|^2 \frac{|x-y|}{2} dx dy$$

subject to the above conditions. (Here $\psi(x) = \int_0^P \tilde{\psi}(p) e^{ipx} \frac{dp}{2\pi}$ is the wave-function in position space.) This variational principle was derived from QCD through QHD in Ref. [8] as well as from an interacting parton model in Ref. [9]. It describes partons (within the mean field approximation) which are interacting with each other through a linear Coulomb potential, which binds them into a baryon. The linear potential comes from eliminating the gluon fields: it is the one-dimensional Fourier transform of the gluon propagator.

A Lorentz invariant formulation (which is more convenient for our purposes) would be to minimize the mass² of the baryon rather than its energy:

$$\mathcal{M}^2 = \left[\int \frac{p}{2} |\tilde{\psi}(p)|^2 \frac{dp}{2\pi} \right] \left[\int_0^P \frac{\mu^2}{2p} |\tilde{\psi}(p)|^2 \frac{dp}{2\pi} + \frac{\tilde{g}^2}{2} \int |\psi(x)|^2 |\psi(y)|^2 \frac{|x-y|}{2} dx dy \right].$$

We are ignoring the flavor and spin quantum numbers of the partons so what we get will be the spin and flavor averaged wavefunction. Also $\mu^2 = m^2 - \frac{\tilde{g}^2}{\pi}$ is the quark mass² after a finite renormalization [12], and m is the current quark mass. The dimensional parameter $\tilde{g} \sim \Lambda_{QCD}$ determines the strength of the interaction. We now define the number density of valence partons

$$V(x_B) = N_c \left[1 + C_1 \left(\frac{\alpha_s(Q_0^2)}{\pi} \right) + C_2 \left(\frac{\alpha_s(Q_0^2)}{\pi} \right)^2 + C_3 \left(\frac{\alpha_s(Q_0^2)}{\pi} \right)^3 \right] \frac{P}{2\pi} |\tilde{\psi}(x_B P)|^2$$

We have normalized this so that the Gross-Llewellyn-Smith sum rule [11] (including the perturbative corrections up to order $\alpha_s^3(Q_0^2)$) is satisfied. The coefficients C_1, C_2 and C_3 are given in Ref. [11]. We will finally set the number of colors N_c to 3 and the number of flavours to 2 at the initial low value of Q_0^2 .

Since the total momentum scales like $P \sim N_c$, in the limit as $N_c \rightarrow \infty$, the parton momentum has the range $0 \leq p < \infty$. In the limit when $N_c \rightarrow \infty$ and $m = 0$, we have found an *exact* minimum of the variational principle: $\tilde{\psi}(p) = Ce^{-\frac{p}{g}}$. The condition for minimizing \mathcal{M}^2 is an integral equation and we can verify that this function is a solution by explicit computation. (The calculation involves infrared singular integrals, which are defined through an appropriate principal value prescription as in [9].) This suggests that even when N_c is finite, a reasonable variational ansatz would be $\tilde{\psi}(p) = C \left(\frac{p}{g}\right)^a \left[1 - \frac{p}{P}\right]^b$. (Recall that $e^{-x} = \lim_{n \rightarrow \infty} [1 - \frac{x}{n}]^n$.) The constant C is determined by the normalization condition. By using the momentum sum rule, we get $b = \frac{N_c}{2f} - 1 + a[\frac{N_c}{f} - 1]$. “ a ” is determined by the variational principle. It is zero in the limit of chiral symmetry and rises like a power of $\frac{m^2}{g^2}$. In the limit $N_c \rightarrow \infty$, a is determined by the transcendental equation

$$\frac{\pi m^2}{\tilde{g}^2} = 1 + \int_0^1 \frac{dy}{y^2} \left[(1+y)^a + (1-y)^a - 2 \right] + \int_1^\infty \frac{dy}{y^2} \left[(1+y)^a - 2 \right]$$

which we derived by a Frobenius-type analysis of the integral equation for the minimization of \mathcal{M}^2 in an earlier paper [9]. Thus in the limit $m = 0$, $a = 0$, and we have the valence parton distribution function $V(x_B) = C[1-x_B]^{\frac{N_c}{f}-2}$ where C is fixed by the GLS sum rule. This variational approximation agrees well with our numerical solution of the same problem in Ref. [9] but is much simpler to use. The limit of chiral symmetry is a good approximation in the case of the nucleon since the current quark masses of the up and down quarks (5-8 MeV) are small compared to the energy scale of the strong interactions ($\Lambda_{QCD} \sim 200$ MeV). The valence parton distribution depends on \tilde{g} only through the ratio $\frac{m^2}{g^2}$ and in the limit $m = 0$, is independent of \tilde{g} .

The structure function $F_3(x_B, Q^2)$ of neutrino scattering on an isoscalar target has been accurately measured by the CCFR and CDHS [10] experimental groups at several values of x_B and Q^2 . To compare with that data we need to evolve our computed distribution function to the appropriate values of Q^2 by the DGLAP equation:

$$\frac{dV(x_B, t)}{dt} = \frac{\alpha_s(t)}{2\pi} \int_{x_B}^1 \frac{dy}{y} V(y, t) P_{qq}\left(\frac{x_B}{y}\right),$$

where $t = \log(Q^2/Q_0^2)$, $\alpha_s(t)$ is evaluated using the two-loop β function [5] and P_{qq} is the evolution kernel to leading order given in Ref. [3]. We solve the evolution equation numerically, assuming an initial value of $Q_0^2 = 0.4 \text{ GeV}^2$. This low value of Q_0^2 is justified since we can show [7] that the anti-quark distribution function of the nucleon is quite small. The range of Q over which we are evolving is small ($Q \sim 0.6$ to 5 GeV) and its effect is small, thus justifying the use of the leading order DGLAP equation. We also set $N_c = 3$, $\Lambda_{QCD} = 200 \text{ MeV}$ and the current quark mass $m = 0$. As for the fraction of baryon momentum carried by valence partons f , we choose it to be 0.5, which is in agreement with the phenomenological fits of parton distribution functions (See e.g., [13].) In a later paper we will derive the gluon structure function as well, and at that time we will have a theoretical prediction for this parameter. The plots show that we have quite good agreement with

data. Thus we have established that Quantum HadronDynamics is a successful way of deriving hadronic structure functions from QCD.

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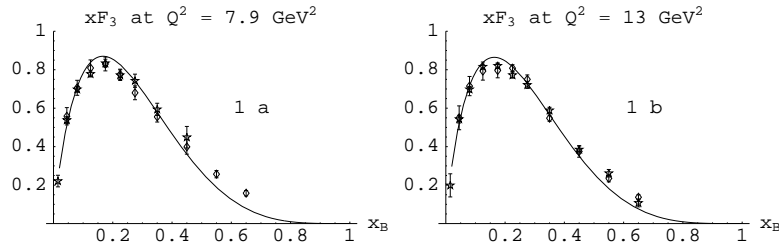


Figure 1: Comparison of QHD prediction of xF_3 (solid curve) with measurements by CCFR(★) and CDHS(◇). We have chosen the parameters $Q_0^2 = 0.4 \text{ GeV}^2$ and $f = 0.5$. (a) CCFR at $Q^2 = 7.9 \text{ GeV}^2$, CDHS at $7.13 \leq Q^2 \leq 8.46 \text{ GeV}^2$ and QHD prediction at $Q^2 = 7.9 \text{ GeV}^2$. (b) CCFR at $Q^2 = 12.6 \text{ GeV}^2$, CDHS at $12.05 \leq Q^2 \leq 14.3 \text{ GeV}^2$ and QHD prediction at $Q^2 = 13 \text{ GeV}^2$.

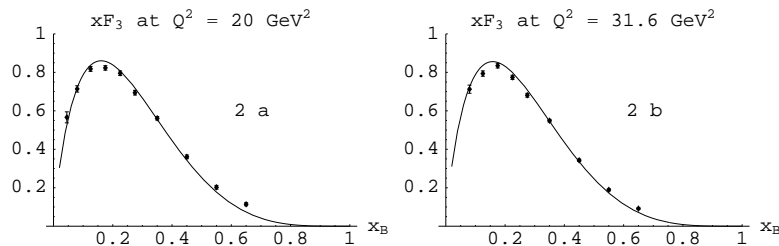


Figure 2: Comparison of QHD prediction of xF_3 (solid curve) with CCFR(●) measurements at (a) $Q^2 = 20 \text{ GeV}^2$ and (b) $Q^2 = 31.6 \text{ GeV}^2$.

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